Risk-adjusted performance measures and implied risk-attitudes

Plantinga, Auke; Groot, Sebastiaan de

RISK-ADJUSTED PERFORMANCE MEASURES AND IMPLIED RISK-ATTITUDES

Auke Plantinga and Sebastiaan de Groot[1]

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Abstract

In this article we study the relation between performance measures and preferences functions. In particular, we examine to what extent performance measures can be used as alternatives for preference functions. We study the Sharpe ratio, Sharpe's alpha, the expected return measure, the Sortino ratio, the Fouse index, and the upside potential ratio. We find that the first three measures correspond to the preferences of investors with a low degree of risk aversion, whereas the latter three measures correspond to the preferences of investors with intermediate and high degrees of risk aversion.

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[1] Auke Plantinga is associate professor of finance at the University of Groningen. Sebastiaan de Groot is an investment analyst at ACAM Advisors LLC, a hedge fund of funds manager in New York. The authors acknowledge the helpful comments from Frank Sortino and Elmer Sterken. The usual disclaimer applies.

1 Introduction

Risk-adjusted performance measures are frequently used to rank investment opportunities. For example, ranking mutual funds is a popular tool in assisting investors with their investment choices, and these rankings are often based on risk-adjusted performance measures. The suggestion implicit in such a ranking is that the first fund is the best fund. The ranking methods differ due to different choices for the return and risk measures as well as the way of adjusting for risk. The justification of a ranking depends on the rationale behind the ranking criterion. For example, a ranking based on the mean geometric return ranks investment opportunities according to the increase in wealth of the investor. Such a measure can be motivated by assuming that the investor wants to maximize future wealth, regardless of risk as measured by volatility. Risk-adjusted performance measures can be motivated by assuming that investors are risk averse and need to be compensated for being exposed to risk. Therefore, the choice of the appropriate performance measure should be determined, at least partially, by the preference function of the investor.

The choice of a performance measure may also be justified by other considerations. A frequently used justification of a performance measure is its ability to identify the investment skills of portfolio managers. Two interesting contributions to this discussion are Dybvig and Ross [1985] and Kothari and Warner [2001]. Both studies focused on the ability of several risk-adjusted performance measures, such as Jensen's alpha and the Sharpe ratio, to identify investment skills. The model of Dybvig and Ross expresses the performance measure as a function of the forecasting skills of the portfolio manager, the standard deviation of returns, and the risk aversion of the investor. Both studies concluded that the performance measures have significant difficulties in detecting investment skills. In order to detect investment skills at the usual levels of significance, the forecasting skills of the manager have to be very significant. This leads to the interesting observation that if a risk-adjusted performance measure is not able to detect forecasting skills in a reliable way, a ranking based on such a measure is not likely to be a ranking of forecasting abilities. Therefore, an alternative justification for the use of risk-adjusted performance measures is necessary.

In this study, we investigate such an alternative justification. In particular, we examine the use of a risk-adjusted performance measure as an alternative to a preference function, such as a utility function or a prospect theory value function. From the perspective of an individual investor, a risk-adjusted performance measure can be regarded an attractive substitute for the preference function of the investor. The construction of a formal preference function may be infeasible for an individual investor due to a lack of mathematical skills or the effort needed to perform such a task. A ranking based on a risk-adjusted performance measure, as published in popular investment magazines or on the web-sites of data vendors such as Morningstar or Micropal, may save the individual a lot of effort and time. However, before a risk-adjusted performance measure can be used this way, the individual should be aware of his risk attitudes in general terms (high or low risk aversion). Furthermore, he should also be aware of the risk attitudes implicit in the use of the performance measures.

Usually, individual investors do not make extensive efforts to formulate formal preference functions of their own behavior. Often, they rely on the help of financial planners, who assist their clients in identifying their risk attitudes. Investors may use, for example, questionnaires focussing on their behavior in hypothetical risky choices, life-style factors, or other factors affecting risk attitudes. However, there is a large group of individuals that do not rely on the help of financial advisors at all. Instead, they may seek help from popular financial magazines or other sources of information on mutual funds, such as Morningstar or Micropal. These sources typically provide investors with rankings of mutual funds based on risk-adjusted performance measures. A risk-adjusted performance measure generally corrects the average return of a mutual fund for the level of risk. The risk-adjustment procedure is an implicit way of modeling risk attitudes. An individual using a ranking based on such a measure adheres to the risk attitudes implicit in the performance measure. Therefore, by using this risk-adjusted performance measure, the individual sacrifices the opportunity to implement his individual risk attitudes. Consequently, it is important to study the implied risk attitudes of these performance measures. In doing so, we try to achieve a general classification of risk-adjusted performance measures into those that correspond to a low degree of risk aversion and those that correspond to a high degree of risk aversion. Using a performance measure rather than a preference function also simplifies the problem of an investor who does not want to model his formal preference function. Such an investor only needs to calibrate his risk preferences in terms of a high versus a low level of risk aversion and select the appropriate ranking device.

Summarizing, the objective of this article is to find the risk preferences implicit in using risk-adjusted performance measures. To this end we take a pragmatic approach. We calculate rankings motivated both by preference functions and risk-adjusted performance measures, and we use rank-correlation coefficients to evaluate the degree of correspondence. In an earlier study, we studied a similar question[2]. We found that some performance measures, such as the Sharpe ratio, are associated with a low level of risk aversion, whereas other performance measures, such as the upside potential ratio and the Fouse index, are associated with a high level of risk aversion. These results were based on a sample of Dutch mutual funds for the period March 1993 through March 1999. In this article we extend this study by using a different data set. This allows us to investigate the sensitivity of our earlier results to changes in the choice of the data set. In addition, we extend the set of preference functions by including the power utility function, which exhibits constant relative risk aversion. This utility function is quite popular, in particular in studies of the equity premium puzzle[3].

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[2] See Plantinga and De Groot [2001].
[3] See, for example, Kocherlakota [1996].

2 Risk-adjusted performance measures based on standard deviation

As stated before, the focus of this study is on the use of risk-adjusted performance measures as an alternative for a preference function in selecting investment opportunities. Examples of such performance measures, which will be discussed subsequently, are the Sharpe ratio, Sharpe's alpha, the Sortino ratio, the Fouse index, and the upside potential ratio.

The Sharpe ratio was introduced as the reward to variability ratio[4] in order to evaluate the performance of mutual funds. The Sharpe ratio is defined as follows:

$$S = \frac{E[R] - R_f}{\sigma}, \qquad (1)$$

where $E[R]$ is the expected rate of return, R_f is the risk-free rate, and σ the standard deviation.

The Sharpe ratio has some attractive features that contributed to its popularity. An important property is that the Sharpe ratio can be used as the objective function in mean-variance optimization[5], where the portfolio with the highest Sharpe ratio is the optimal portfolio of risky assets. Consequently, the Sharpe ratio is an obvious choice to be included in this study.

Sharpe [1994] pointed out that the Sharpe ratio can be interpreted as a t-statistic to test the hypothesis that the return on the portfolio is equal to the risk-free return. A higher Sharpe ratio is consistent with a higher probability that the portfolio return will exceed the risk-free return. Consequently, the Sharpe ratio can be used by investors who prefer a portfolio with a minimal probability of falling below the risk-free rate.

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[4] See Sharpe [1966].

Similar to mean-variance theory, the Sharpe ratio is motivated by either of the following two assumptions:

1. Returns are normally distributed;
2. The investor has a preference function in terms of mean and variance.

Based on these assumptions, alternative performance measure can be derived. Consider the following performance measure, which we will call Sharpe's alpha:

$$\alpha = E[R] - A\sigma^2 , \tag{2}$$

where A is a parameter driving the level of risk aversion. This measure is often used as an alternative representation of the quadratic utility function.

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[5] See Elton and Gruber [1995] and Benninga [1997].

3 Risk-adjusted performance measures based on downside risk

In addition to the Sharpe ratio and Sharpe's alpha, this study also examines several risk-adjusted performance measures that are not based on the aforementioned assumptions. A common characteristic of these alternative performance measures is the use of so-called downside deviation with respect to a reference point. The reference point, which may also be called the minimal acceptable rate of return, is used to distinguish "risk" from "volatility". According to Sortino and Van der Meer [1991], realizations above the reference point imply that goals are accomplished and, therefore, are considered "good volatility". Realizations below the reference point imply failure to accomplish the goals and should be considered "bad volatility" or risk. Based on this premise, this study investigates the Sortino ratio, the Fouse-index, and the upside-potential ratio. The Sortino ratio is probably the most well-known measure, and it is calculated as follows:

$$Sort = \frac{E[R] - R_{mar}}{\delta}, \tag{3}$$

where R_{mar} is the minimal acceptable rate of return and δ is the downside risk with respect to the minimal acceptable rate of return.

The Fouse index is the equivalent of Sharpe's alpha in a mean – downside risk environment. Sortino and Price [1994] defined the measure as follows:

$$Fouse = E[R] - B\delta^2, \tag{4}$$

where B is a parameter representing the degree of risk aversion of the investor.

The Sortino ratio and the Fouse index rely on the use of expected return and downside risk. Expected return is used as a measure of the potential reward of an investment opportunity. An alternative for using the expected return is the so-called upside potential ratio, which is the probability weighted average of returns above the reference rate. The upside potential ratio was developed by Sortino, Van der Meer, and Plantinga [1999] and is defined as:

$$UPR = \frac{\sum_{t=1}^{T} \iota^{+} \frac{1}{T}\left(R_t - R_{mar}\right)}{\sum_{t=1}^{T} \iota^{-} \frac{1}{T}\left(R_t - R_{mar}\right)^2} \tag{5}$$

where T is the number of periods in the sample, R_t is the return of an investment in period t, $\iota^{+}=1$ if $R_t>R_{mar}$, $\iota^{+}=0$ if $R_t \leq R_{mar}$, $\iota^{-}=1$ if $R_t \leq R_{mar}$ and $\iota^{-}=0$ if $R_t>R_{mar}$. An important advantage of using the upside potential ratio rather than the Sortino ratio is the consistency in the use of the reference rate for evaluating both profits and losses.

Finally, an important difference between downside risk and standard deviation is the use of an exogenous reference rate versus the mean return. The investor's objective function motivates the choice of the reference rate. As a result, a part of the investor's preference function is introduced into the risk calculation. Therefore, the resulting calculation is only valid for individuals sharing the same reference rate. Investors with different minimal acceptable rates of return will have different rankings.

4 Preference functions

Economists usually represent the preferences of individuals by using a preference function, a mathematical function that enables the individual to rank relevant choices. As all the performance measures discussed in the previous sections are also mathematical functions that can be used to rank choices, this definition implies that all these measures are also preference functions. Consequently, the distinction between performance measures and preference functions is somewhat arbitrary and is largely based on their origination in the relevant literature. Performance measures are usually proposed in the literature on selecting investment opportunities and mutual funds, whereas preference functions are associated with the literature on modeling (hypothetical) choices by individuals.

Parallel to the discussion on the choice of the appropriate performance measure is a similar discussion on the choice of the appropriate preference function. This study focuses on the discussion regarding the choice between a normative utility function and a descriptive preference function.

In the classical economic theory, utility functions are the favored kind of preference functions. Utility functions model the subjective risk attitudes of the individual investor. Consequently, individual investors may differ in their degree of risk aversion: one investor can be extremely risk averse, whereas another can be less risk averse. Usually, investors are assumed to be risk averse. Utility functions are a special class of preference function that satisfy a set of axioms guaranteeing that the individual exhibits consistent and rational behavior[6]. Even within the class of utility functions, a wide variety of possible functional forms are available, each with different characteristics. Often, the choice of the utility function in an economic model seems to be driven by the capacity to generate analytically tractable solutions. For example, in portfolio optimization applications, the quadratic utility function is often used as it closely corresponds to this type of problem: it can be expressed directly in terms of the parameters that reflect the expectation and standard deviation of the return distribution of the investment opportunities. The fact that the quadratic utility function has the undesirable property of decreasing utility at a sufficiently high level of wealth seems to be ignored and can be considered to be the price of building a simple model.

The quadratic utility function is defined as:

$$U(x) = x - k x^2, \tag{6}$$

where x represents the wealth level, and k is a parameter driving the risk aversion of the investor. A relevant property of a utility function is the behavior of relative risk aversion as a function of wealth[7]. An investor with a quadratic utility function displays increasing relative risk aversion, which implies that the investor tends to invest less in risky assets as his wealth increases. Increasing relative risk aversion does not seem to be a very plausible assumption. For example, Blume and Friend [1975] found evidence in support of constant relative risk aversion (CRRA), which implies that the relative allocation to risky assets is not affected by the level of wealth. The following power utility function exhibits CRRA and is frequently used in the literature:

$$U(x) = \frac{w^{1-\lambda}}{1-\lambda}, \tag{7}$$

where λ is a parameter driving the degree of risk aversion of the investor. Many studies suggest that λ should be in the range between 1 and 2[8].

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[6] See Fama and Miller [1972].
[7] See Pratt [1964].
[8] Kocherlakota [1996] presented a discussion on reasonable values for λ.

Empirical studies showed that the behavior implied in using utility functions may be inconsistent with real behavior. For example, Tversky and Kahneman [1992] found that investors display risk-seeking behavior in choices involving losses and risk-averse behavior in choices involving gains. Tversky and Kahneman developed the so-called prospect theory value function that facilitates both risk-seeking and risk-averse behavior. The prospect theory value function is a descriptive preference function that does not satisfy the axioms of rational behavior and, therefore, is not a utility function. Descriptive preference functions are evaluated based on their ability to explain actual decisions of individuals, whereas more traditional research usually focuses on normative choice behavior in experimental settings.

A distinctive property of the prospect theory value function is that risk attitudes change on either side of a reference point. Tversky and Kahneman suggested that the reference point is equal to the current wealth level. In the domain of outcomes below the current wealth level (the so-called domain of losses), the investor exhibits a preference for risk. On the other hand, in the domain of outcomes above the current wealth level, the so-called domain of gains, the investor exhibits risk aversion. Tversky and Kahneman suggested the following specification of the prospect theory value function:

$$v(x) = \left\langle \begin{array}{ll} x^{\alpha} & x \geq 0 \\ -k(-x)^{\beta} & x < 0 \end{array} \right. , \tag{8}$$

where k is a constant reflecting the concept of loss aversion, and the parameters α and β are related to risk attitudes and determine the shape of the function. Based on empirical research, Tversky and Kahneman [1992] found the following parameter estimates: α=β =0.88 and k=2.25. As an alternative for the coefficients suggested by Tversky and Kahneman, this study also examined a piecewise linear version of the prospect theory value function, where $\alpha = \beta = 1$.

In order to use the prospect theory value function in a way that is consistent with the performance measures based on downside risk, we choose a minimal acceptable rate of return equal to 0%. More generalized versions of the prospect theory value functions allow for reference points different from 0%[9].

[9] See, for example, De Groot [1998].

5 Comparing risk-adjusted performance measures and preference functions

This section answers the main question raised in this paper, as it evaluates the risk preferences implicit in risk-adjusted performance measures. In addition, this section investigates the quality of risk-adjusted performance measures as an alternative to preference functions. To be suited as an alternative to a preference function, a risk-adjusted performance measure should facilitate the trade-off between risk and return. Given the concept of risk aversion, it is reasonable to expect that funds with a high level of risk, however that may be measured, should be penalized.

A first analysis of the trade-off between risk and return is to examine the rank-correlation coefficients between the risk-adjusted performance measure and the risk and return measures. To this end, we collected a sample of 253 U.S. mutual funds returns from Datastream. Each mutual fund has a return history of 312 months. In Table 1 we present some general statistics for the funds included in our sample. We estimated the investment styles using Sharpe's style regressions[10]. We obtained data on a bond index and IIA Style indices for U.S. Value, U.S. growth, European Value, European Growth, Pacific Value, and Pacific Growth stocks from Micropal and used these as explanatory variables in the style regression. A fund is classified as having a particular style if the style coefficient for the corresponding index exceeds 50%. The majority of the funds has only style exposure to domestic (regional) factors. Except for 42 bond funds, all funds can be classified as either a value fund or a growth fund.

[10] See Sharpe [1992].

Table 1: Performance and style characteristics of the funds

	# funds	E[R]	σ	R2
All funds	253	0.56%	5.79%	53.70%
Bond style	42	0.32%	2.50%	57.12%
Value style	75	0.61%	6.06%	52.90%
Growth style	107	0.62%	7.12%	51.80%
Unclassified	29	0.58%	4.96%	57.80%
Domestic style	246	0.55%	5.45%	54.76%

Value, growth and domestic funds are classified as such if the exposure to the appropriate style factor exceeds 50%. Funds without an exposure over 50% to any of the style factors are unclassified.

In Table 2 we present the association (as measured by rank-correlation coefficients) between the rankings based on different performance measures, expected return, standard deviation, and downside risk. For each of the 253 funds in the sample, we calculated the value of the Sharpe ratio, the Sortino ratio, Sharpe's alpha, the Fouse index, and the upside potential ratio based on observed monthly return data. The return on 1-month treasury bills is used to approximate the risk-free rate in the calculation of the Sharpe ratio. For the Sortino ratio, the Fouse index, and the upside potential ratio, we set the minimal acceptable rate of return equal to 0%. The risk aversion parameter used in Sharpe's alpha and the Fouse index is set equal to 1.

Table 2: Rank-correlation coefficients of performance and risk measures.

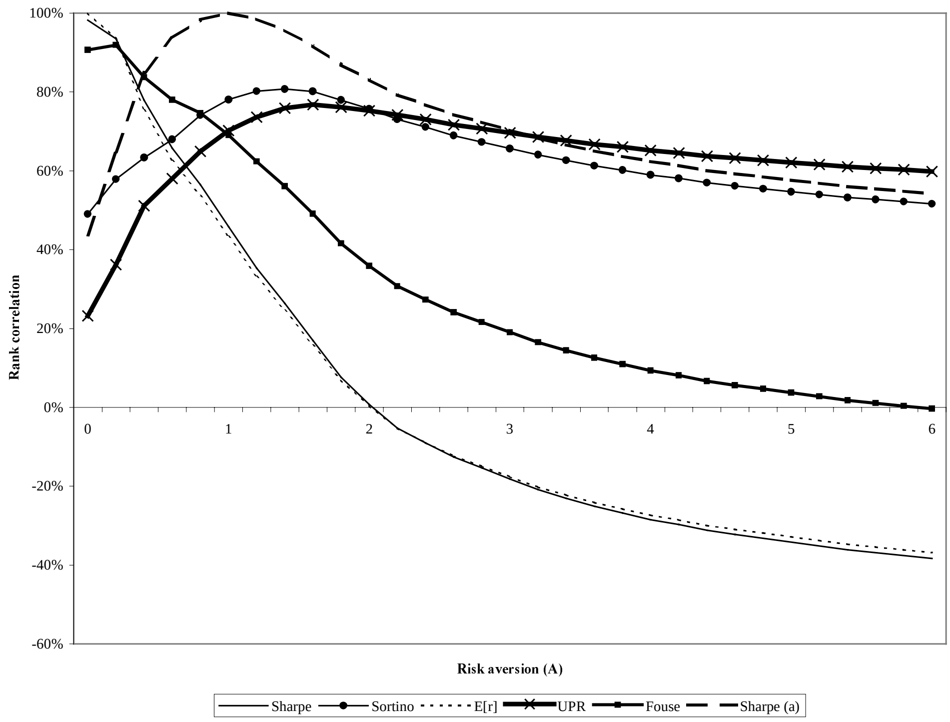
	Sharpe	Sortino	Sharpe α	Fouse	UPR
Sharpe	100.00%	44.81%	46.28%	88.17%	21.16%
Sortino	44.81%	100.00%	77.98%	76.67%	82.75%
Sharpe α	46.28%	77.98%	100.00%	69.26%	70.07%
Fouse	88.17%	76.67%	69.26%	100.00%	50.54%
UPR	21.16%	82.75%	70.07%	50.54%	100.00%
E[r]	98.20%	49.02%	43.73%	90.66%	23.19%
Std	51.74%	-38.55%	-41.29%	14.72%	-50.08%
Dwnsr	43.18%	-48.53%	-48.77%	4.50%	-61.13%

Rank-correlation coefficients calculated based on monthly returns from January 1975 to June 2001.

Table 2 shows that all risk-adjusted performance measures are positively correlated with each other. The correlation varies from a low of 21.2% (Sharpe ratio and upside potential ratio) to a high of 88.2% (Sharpe ratio and Fouse index). There is no obvious pattern regarding differences between measures based on standard deviation and measures based on downside risk. One might expect that measures based on the same risk measure should be highly correlated while measures based on different risk measures should have a low correlation. However, table 2 seems to indicate that this hypothesis does not hold. For example, the Sharpe ratio, which is based on standard deviation, correlates more with the Fouse index (which relies on a different risk measure, downside risk) than with Sharpe's alpha (which relies on the same standard deviation). Furthermore, it should be noted that both the Sharpe ratio and the Fouse index are highly correlated with expected return.

It is also worthwhile to look at the correlation between the risk-adjusted performance measures and the risk measures itself. An interesting observation is that the Sharpe ratio shows a considerable positive correlation with both risk measures. In other words, high Sharpe ratios are associated with high risk levels. The Fouse index also shows a positive but negligible correlation with both risk measures. The Sortino ratio, Sharpe's alpha, and the upside potential ratio show negative correlation with the risk measures[11].

Figure 1: Rank correlation between risk-adjusted performance measures and quadratic utility function



The degree of correspondence between a preference function and the risk-adjusted performance measures can be measured by the rank-correlation coefficient. Figure 1 displays the rank-correlation coefficient as a function of the parameter k of the quadratic utility function specified in equation (6). The most prominent observation is that no performance measure globally dominates in terms of rank

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[11] It should be noted that the correlation coefficients for the Fouse index depend on the choice of the risk-aversion parameter. The higher the parameter for risk aversion, the higher the

correlation with the quadratic utility function. For low values of k, corresponding with low levels of risk aversion, the Sharpe ratio, the Fouse index, and the expected rate of return correlate well with the utility function. However, for higher values of k, corresponding with high levels of risk aversion, correlation with the preference function is falling rapidly and even becomes negative. The upside potential ratio, Sharpe's alpha, and the Sortino ratio display reasonable results, except for very low values of the risk-aversion parameter k. However, the correlation between these measures and the risk-aversion coefficient is not constant: for low values of k correlation is low, for intermediate values (around $k = 1$) correlation is very high, and for high values of k correlation is dropping slowly. Nevertheless, for high values of k, these measures provide a far better approximation of the investor's preferences than the Sharpe ratio, the Sortino ratio, and the expected return measure.

These results are consistent with Jia and Dyer [1996]. Jia and Dyer showed that the quadratic utility function is one of two classes of continuously differentiable functions that can be represented in the form of a separable risk-value model. In other words, the expected value of a quadratic utility function can be rewritten in terms of return and standard deviation as follows:

$$E[U(W)] = U(E[R]) - k\sigma_r^2 . \tag{9}$$

Since Sharpe's alpha has a form similar to equation (9), there must be a quadratic utility function that generates the same ranking. Since none of the other performance measures has a functional form similar to equation (9), it should not be expected that any of these measures would correlate perfectly with the quadratic utility function.

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importance of the risk measure in determining the ranking correlation.

Figure 2: Rank correlation between risk-adjusted performance measures and prospect theory value functions

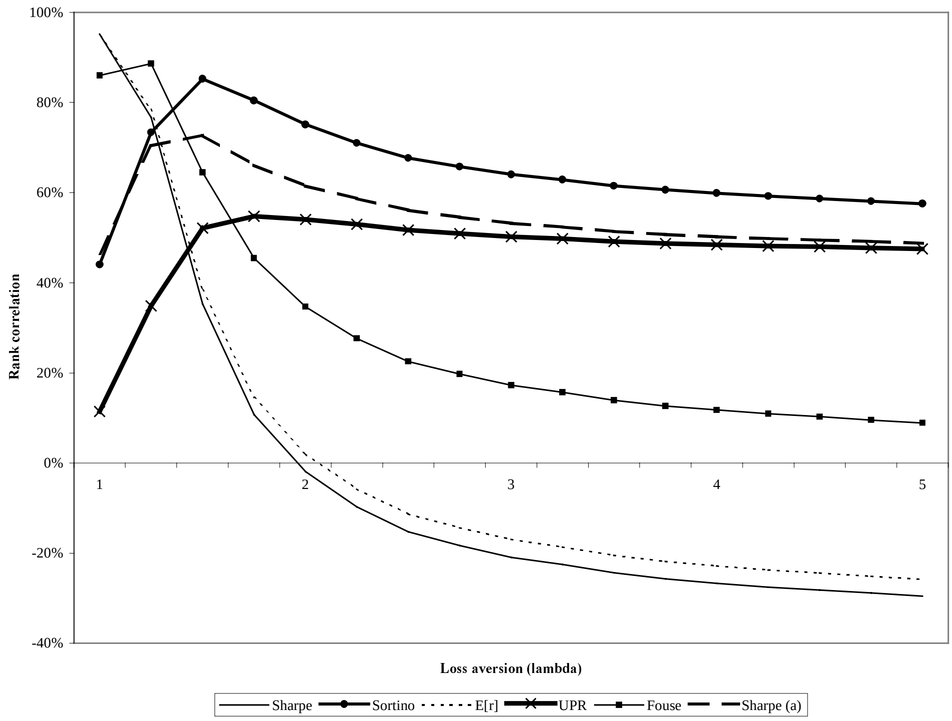

Figure 2 displays the rank correlation between the risk-adjusted performance measures and several prospect theory value functions while varying the parameter *k*, the degree of loss aversion. Consistent with the results for the quadratic utility function, we find that the Sharpe ratio, the Fouse index, and the expected return measure closely represent the preferences of investors with a low degree of loss aversion. As with the quadratic utility function, correlation is falling rapidly when increasing the coefficient of loss aversion. For higher levels of loss aversion, the Sortino ratio yields the best results with a correlation of approximately 60% with the preference function. Sharpe's alpha and the upside potential ratio give reasonable results with a correlation of approximately 50%. An analysis using a piecewise linear value function with $\alpha=\beta=1$ shows similar patterns.

Figure 3: Rank correlation between risk-adjusted performance measures and power utility functions

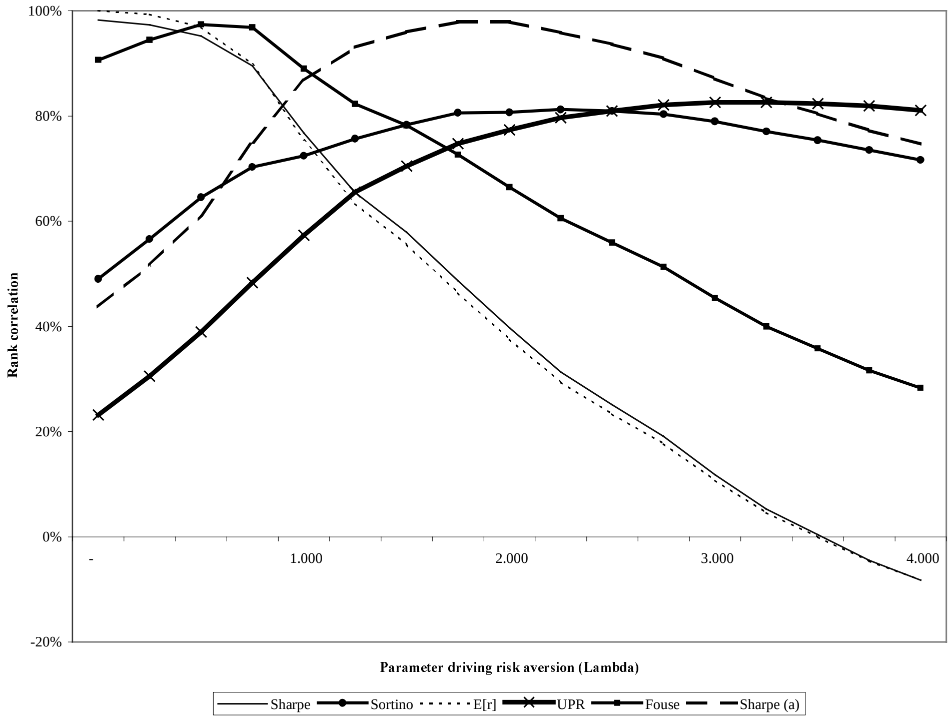

Figure 3 displays the rank correlation between the risk-adjusted performance measures and power utility functions. Consistent with previous results, we find that for the Sharpe ratio, the Fouse index, and the expected return measure, the rank-correlation coefficients decrease with an increase in the level of risk aversion. For intermediate levels of risk aversion, around $\lambda = 2$, Sharpe's alpha dominates all other measures, whereas for high levels of risk aversion the upside potential ratio dominates. As stated before, empirical studies suggest that realistic values of λ are in the interval between 1 and 2. This implies that investors could use Sharpe's alpha with a risk aversion parameter equal to 1.

6 Conclusions

In this article we investigated the risk preferences implicit in the use of risk-adjusted performance measures. To this end, we studied mutual fund rankings from the perspective of an individual investor who wants to invest in a mutual fund. In particular, this investor has to select one mutual fund from a large universe of funds. We analyze the differences between the outcomes of rankings based on performance measures and rankings based on preference functions. Both approaches to select investment opportunities have similar data requirements. For each investment opportunity, both approaches require a return distribution. In addition, preference functions need to be calibrated to reflect the proper risk attitudes of the individual.

We find that each risk-adjusted performance measure can be associated with a different level of risk aversion. Furthermore, we find a pattern that is consistent among several classes of preference functions. The Sharpe ratio, the Fouse index, and the expected return measure can be associated with low levels of risk and/or loss aversion. However, these measures display a diminishing correlation between the preference function and the level of risk (loss) aversion. Therefore, investors displaying a sufficiently high level of risk and/or loss aversion, should use a ranking based on either the Sharpe ratio, the Fouse index, or the expected return measure.

The results for Sharpe's alpha, the Sortino ratio, and the upside potential ratio imply that for low levels of risk (loss) aversion, these measures do not represent the preferences of investors. The best results are generated for investors with intermediate levels of risk aversion. Nevertheless, even for high levels of risk aversion, these measures dominate the outcomes for the Sharpe ratio, the Fouse index, and the expected return measure. Finally, this study confirms the results of an earlier study (Plantinga and De Groot [2001]), which was based on the returns of European mutual funds from 1993 to 1999.

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